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A RANDOM NUMBER GENERATOR FOR CONTINUOUS RANDOM VARIABLES BASED ON AN INTERPOLATION PROCEDURE OF AKIMA

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ABSTRACT

A Fortran-IV subroutine is given which may be used to generate random observations of a continuous random variable from a table of discrete observations. This subroutine employs Akima's algorithm for cubic spline interpolation.

The compatibility of problem and algorithm is such that this simple routine gives very good results.

1. The Algorithm. An important part of any simulation study is the sampling of a random variable x with cummulative distribution function $F(x)$. Often F is specified only by a table of $(x,F(x))$, the probability distribution of the random variable $y=F(x)$ is the uniform distribution $U(0,1)$. There are satisfactory pseudorandom number generators to sample from $U(0,1)$. For example from Knuth [2]

$$(1.1)\quad y_{n+1} = 31415926y_n + 27182818 \pmod{2^{35}}.$$

The problem of sampling from $F(x)$ may be looked at, then, as equivalent to finding an inversion formula

$$(1.2)\quad x = F^{-1}(y).$$

To approximate $F^{-1}(y)$ we use an empirically obtained table

$$(1.3)\quad \{(x_1,F(x_1)): x_1 < x_2 < \ldots < x_k\}$$

plus a good interpolation routine. We should have in (1.3) that $F(x_1) \sim 0$ and $F(x_k) \sim 1$. In this way it is possible to take random values from $U(0,1)$ and obtain their approximate inverses as random samples from "nearly $F(x)$."

The sampling problem now reduces to one of interpolation subject to a montonicity constraint or "isotone interpolation." Neither the standard polynomial interpolation nor the cubic spline interpolation have a chance of accomplishing this objective. For this reason (and others) the standard approach in the literature is piecewise linear interpolation. Observe that piecewise liner interpolation maintains monotonicity but, in general, at the expense of accuracy.

It is the purpose of this note merely to point out that Akima's [1] quasi-Hermite piecewise cubic interpolation should be very effective on this problem; since it was designed to handle similar problems. A random number generator (RNG) is advocated which uses Akima interpolation. The user reads in a table

$$\{(x_{(i)},F(x_{(i)})): x_{(0)} < x_{(1)} < \ldots < x_{(k)} < x_{(k+1)}\}.$$

The routine then computes and stores the coefficients of $x = F^{-1}(y)$. Calls on the random number generator will first obtain a pseudorandom number y from $U(0,1)$. If $y \le F(x_{(1)})$, the random number given is $x_{(1)}$, if $y \ge F(x_{(k)})$, the random number given is $x_{(k)}$. If $F(x_{(1)}) < y < F(x_{(k)})$, the random number given

is $F^{-1}(y)$. A listing of the random number generator is given in Appendix II.

Naturally, the quality of the random number generator proposed is determined by the ability of F^{-1} to approximate a variety of cumulative distribution functions. In Appendix I, we show how the interpolation errors of $F^{-1}(y)$ compare with those of linear interpolation in the case of the standard normal, Cauchy and double exponential distributions with location 0 and scale 1. Exact values of $F^{-1}(y)$ are assumed given for y between .50 and .98 in increments of .01 and for y between .98 and .995 in increments of .005. Interpolated values of F^{-1} and linear interpolation are obtained at points between the mesh values and the relative errors computed and compared.

REFERENCES

1. Akima, H., A new method of interpolation and smooth curve fitting based on local procedures, J. ACM, 17(1970), 589-602.
2. Knuth, Donald E., Seminumerical Algorithms, Addison-Wesley, Reading, Mass., 1969.

APPENDIX I

NORMAL DISTRIBUTION

F(x)=	x=	E(AKIMAS)=	E(LINEAR)=
0.50333	0.00833	0.00006	0.00011
0.52333	0.05837	0.00003	0.00020
0.54333	0.10858	0.00002	0.00033
0.56333	0.15910	0.00006	0.00044
0.58333	0.21005	0.00004	0.00061
0.60333	0.26157	0.00001	0.00080
0.62333	0.31381	0.00001	0.00096
0.64333	0.36694	0.00000	0.00114
0.66333	0.42114	0.00004	0.00129
0.68333	0.47662	0.00005	0.00149
0.70333	0.53362	0.00004	0.00172
0.72333	0.59241	0.00002	0.00200
0.74333	0.65334	0.00000	0.00231
0.76333	0.71680	0.00007	0.00258
0.78333	0.78329	0.00004	0.00300
0.80333	0.85343	0.00005	0.00346
0.82333	0.92805	0.00007	0.00401
0.84333	1.00823	0.00012	0.00469
0.86333	1.09546	0.00011	0.00561
0.88333	1.19193	0.00018	0.00679
0.90333	1.30097	0.00021	0.00853
0.92333	1.42812	0.00030	0.01115
0.94333	1.58372	0.00034	0.01569
0.96333	1.79115	0.00185	0.02539
0.98333	2.12849	0.00336	0.02805
0.99333	2.47156	0.02432	0.07186

CAUCHY DISTRIBUTION

F(x)=	x=	E(AKIMAS)=	E(LINEAR)=
0.50333	0.01047	0.00002	0.00010
0.52333	0.07344	0.00005	0.00054
0.54333	0.13698	0.00005	0.00098
0.56333	0.20164	0.00005	0.00143
0.58333	0.26795	0.00005	0.00190
0.60333	0.33654	0.00006	0.00237
0.62333	0.40809	0.00006	0.00287
0.64333	0.48342	0.00007	0.00340
0.66333	0.56347	0.00008	0.00396
0.68333	0.64941	0.00009	0.00456
0.70333	0.74267	0.00011	0.00521
0.72333	0.84507	0.00013	0.00592
0.74333	0.95897	0.00015	0.00672
0.76333	1.08749	0.00017	0.00761
0.78333	1.23490	0.00021	0.00865
0.80333	1.40714	0.00025	0.00985
0.82333	1.61283	0.00030	0.01128
0.84333	1.86499	0.00037	0.01304
0.86333	2.18419	0.00046	0.01526
0.88333	2.60509	0.00057	0.01820
0.90333	3.19100	0.00068	0.02229
0.92333	4.07127	0.00072	0.02844
0.94333	5.55776	0.00021	0.03882
0.96333	8.64274	0.00113	0.06036
0.98333	19.08113	0.02053	0.06658
0.99333	47.73947	0.08864	0.16661

DOUBLE EXP DISTRIBUTION

F(x)=	x=	E(AKIMAS)=	E(LINEAR)=
0.50333	2.09951	0.00002	0.00223
0.52333	2.22281	0.00003	0.00233
0.54333	2.35140	0.00002	0.00244
0.56333	2.48575	0.00002	0.00255
0.58333	2.62641	0.00002	0.00268
0.60333	2.77398	0.00003	0.00281
0.62333	2.92918	0.00003	0.00295
0.64333	3.09286	0.00004	0.00312
0.66333	3.26599	0.00003	0.00331
0.68333	3.44972	0.00004	0.00351
0.70333	3.6/544	0.00005	0.00375
0.72333	3.85483	0.00006	0.00402
0.74333	4.07993	0.00006	0.00434
0.76333	4.32331	0.00007	0.00471
0.78333	4.58819	0.00009	0.00514
0.80333	4.87874	0.00010	0.00567
0.82333	5.20047	0.00013	0.00631
0.84333	5.56091	0.00015	0.00712
0.86333	5.97063	0.00019	0.00817
0.88333	6.44530	0.00024	0.00957
0.90333	7.00946	0.00031	0.01157
0.92333	7.70487	0.00040	0.01461
0.94333	8.61171	0.00042	0.01983
0.96333	9.91767	0.00215	0.03088
0.98333	12.28305	0.00450	0.03291
0.99333	15.03195	0.02895	0.00170

APPENDIX II

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C INTERPOLATION OF THE INVERSE NORML  DISTRIBUTION
      COMMON N,P1(50),P2(50),P3(50),P4(50),X(50)
C N IS THE NUMBER OF INTERPOLATING KNOTS. N SHOULD BE LESS OR EQ TO
      N = 50
      CALL INIT
C M IS THE NUMBER OF RANDOM NUMBERS TO BE GENERATED M SMALLER THAN
      M = 20
      CALL RNG(M)
       CALL EXIT
      END

      SUBROUTINE INIT
C ********************
C THIS SUBROUTINE GENERATES THE PARAMETERS OF THE CUBICS THAT
C   INTERPOLATE THE DATA KNOTS
C   ARRAYS P1,P2,P3,P4 CONTAIN THE PARAMETERS
      COMMON N,P1(50),P2(50),P3(50),P4(50),X(50)
100   FORMAT (2F12.6)
200   FORMAT  (2X,10F12.6)
C ******* READ DATA ***************************
      READ 100,((X(I),P4(I)),I=1,N)
C ********** READ DATA FINISHES ***********
      N2 = N-2
      S1=(P4(2)-P4(1))/(X(2)-X(1))
      S2=(P4(3)-P4(2))/(X(3)-X(2))
      S3=(P4(3)-P4(4))/(X(3)-X(4))
      DO 10 I=3,N2
      S4 = (P4(I+2) -P4(I+1))/(X(I+2) - X(I+1))
      W1 = ABS(S2-S1)
      W3 = ABS(S4-S3)
      IF (W1+W3) 20,30,20
20    P3(I) = (W3*S2 + W1*S3)/(W1+W3)
      GO TO 40
30    P3(I) = .5*(S2+S3)
40    S1 = S2
      S2 = S3
10    S3 = S4
      DO 60 I=3,N2
      A= X(I+1) - X(I)
      P1(I) = (P3(I) + P3(I+1) - 2.0*(P4(I+1) - P4(I))/A)/(A*A)
      P2(I)=(3.0*(P4(I+1)-P4(I)) /A - 2.0*P3(I) - P3(I+1))/A
60    CONTINUE
      RETURN
      END

       SUBROUTINE RNG(M)
       COMMON N,P1(50),P2(50),P3(50),P4(50),X(50)
       REAL*8 RN(20),I34,I35
100    FORMAT (I6,9F12.6)
C ****** RANDOM NUMBER GENERATOR ********
       I34 = 57721566
       I35 = 2.0**35
       DO 10 I=1,M
       I34 = ( 31415926  *I34 + 27182828  )
       I34 = (DMOD(I34,I35))
       RN(I) = I34/I35
       PRINT 100,I,RN(I)
       IF (RN(I).LE.X(3)) RN(I) = X(3)
10     IF (RN(I).GE.X(N-2)) RN(I)= X(N-2)
       IC = 1
       N3 = N-3
       DO 20 I =3,N3
       DO 30 J=1,M
       V = RN(J) - X(I)
       IF (RN(J)-X(I)) 40,50,50
50     IF (RN(J) - X(I+1)) 60,60,40
60     YRN  = P1(I)*V*V*V + P2(I)*V*V + P3(I)*V + P4(I)
C  YRN IS THE GENERATED RANDOM OBSERVATION
       PRINT 100,IC,RN(J),YRN
       IC = IC + 1
40     CONTINUE
       IF (IC.GT.M) GO TO 9
30     CONTINUE
20     CONTINUE
9      CONTINUE
       RETURN
       END
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